PHYSICAL REVIEW A

ATOMIC, MOLECULAR, AND OPTICAL PHYSICS

RAPID COMMUNICATIONS

The Rapid Communications section is intended for the accelerated publication of important new results. Since manuscripts submitted to this section are given priority treatment both in the editorial office and in production, authors should explain in their submittal letter why the work justifies this special handling. A Rapid Communication should be no longer than 4 printed pages and must be accompanied by an abstract. Page proofs are sent to authors.

Polarization state of a biphoton: Quantum ternary logic

A. V. Burlakov, M. V. Chekhova, O. A. Karabutova, D. N. Klyshko, and S. P. Kulik
Department of Physics, Moscow State University, 119899 Moscow, Russia
(Received 21 July 1999)

The polarization state of biphoton light generated via colinear frequency-degenerate spontaneous parametric down-conversion is considered. A biphoton is described by a three-component polarization vector, its arbitrary transformations relating to the SU(3) group. A subset of such transformations, available with retardation plates, is realized experimentally. In particular, two biphoton beams, formed by colinearly polarized photons (type I) are transformed into a single biphoton beam formed by orthogonally polarized photons (type II). Polarized biphotons are suggested as ternary analogs of two-state quantum systems (qubits). [S1050-2947(99)51312-7]

PACS number(s): 03.67.−a, 42.50.Ar, 42.50.Dv

The polarization state of a single photon is described by a two-dimensional normalized polarization vector. As any quantum system with two basic states [1], an arbitrarily polarized photon can represent a qubit—a quantum bit of information [2] used in quantum computation. Several quantum logical operations with photon qubits have been proposed, which make use, in addition to photons, of atoms or ions [3]. Recently, quantum gates were suggested based only on photons, some of them serving as polarization qubits and some as location qubits [4].

In this paper, we consider a quantum system formed by two correlated photons—a biphoton emitted via frequency-degenerate colinear spontaneous parametric down-conversion (SPDC). Its polarization state is assumed to be arbitrary. In this general case, the biphoton can be described by the state vector [5]

$$\Psi = c_1|2,0\rangle + c_2|1,1\rangle + c_3|0,2\rangle, \tag{1}$$

where $c_i = d_i e^{i\phi_i}$ are complex amplitudes and the notation $|N_x, N_y\rangle$ means a state with N_x photons in the horizontal (x) polarization mode and N_y photons in the vertical (y) polarization mode, with $N_x + N_y = 2$. The normalization condition is $\Sigma_i |c_i|^2 = 1$. In most cases, the total phase of the state (1) is not essential, so one can assume $\phi_2 = 0$, and the three-component state of a biphoton is given by four real parameters. One can introduce the "polarization vector" of a biphoton,

$$\mathbf{e} = (c_1, c_2, c_3). \tag{2}$$

In the most general form, the state (1) can be prepared via SPDC generated in three nonlinear crystals with common coherent pumping. The states $|2,0\rangle$ and $|0,2\rangle$ are generated via type-I SPDC, and the state $|1,1\rangle$ via type-II SPDC [6]. According to Eq. (1), a biphoton is a three-state system, similar to a particle with spin 1. Arbitrary transformations of polarization vectors $\mathbf{e}$ are given by unitary 3×3 matrices G, $G^\dagger G = I$, $\det G = 1$, which form a three-dimensional representation of the SU(3) group; see [7]. This type of symmetry, which is well known in nuclear physics but seems to be new to optics, could be used for developing "ternary logic" in quantum computation. To each of the three basic states $|2,0\rangle$, $|1,1\rangle$, and $|0,2\rangle$, one can assign one of the digits 0, 1, and 2. The advantage of "ternary" quantum logic over binary logic (qubits) is the larger number of states that can be covered by an n-element quantum register: 3^n instead of 2^n. The first question arising here is, how can one "switch" between these three basic states or their combinations?

According to the properties of the SU(3) group, an arbitrary transformation G of the vector $\mathbf{e}$ is given by eight real parameters. Linear lossless elements (retardation plates and polarization rotators) introduced into the biphoton beam transform the vector $\mathbf{e}$ but cannot give all possible matrices G. A transformation of this kind can be characterized by three independent parameters, and therefore it only realizes a
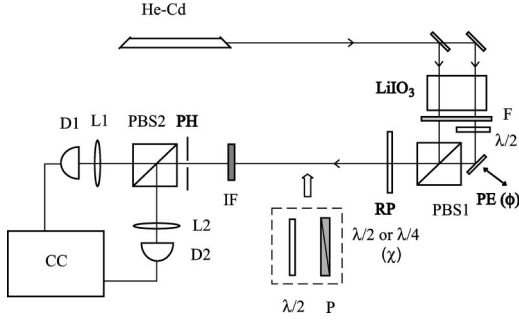
FIG. 1. The experimental setup. cw radiation of He-Cd laser at 325 nm is fed into a Mach-Zehnder interferometer, so that two coherent pump beams excite colinear frequency-degenerate SPDC in different spatial domains of a $LiIO_3$ crystal. The cutoff filter F suppresses the pump radiation, and the $\lambda/2$ plate rotates polarization of the SPDC light in one of the arms by $\pi/2$. The piezoelectric element PE is used for path length (phase ϕ) variation. The polarizing beam splitter PBS1 joins two SPDC beams together. The retardation plate (RP), either $\lambda/2$ or $\lambda/4$, can be rotated by angle χ. The registration part of the setup includes the interference filter IF and the pinhole PH selecting SPDC radiation; the polarizing beam splitter PBS2; two detectors D1, D2; lenses L1, L2, focusing the radiation on the detectors; and the coincidence circuit CC. The framed block, including a polarizer P and a $\lambda/2$ plate, is introduced for measuring G_{xx}, G_{yy}; without this block, G_{xy} is measured.

three-dimensional representation of the SU(2) group, which leaves invariant the polarization degree P [5,8,9]. At the same time, two of the basic states in superposition (1) have $P=1$ and one has $P=0$.

However, by passing from the basis $|2,0\rangle, |1,1\rangle, |0,2\rangle$ to the basis

$$\Psi_+ = \frac{|2,0\rangle + |0,2\rangle}{\sqrt{2}} \equiv |+,-\rangle,$$

$$\Psi_- = \frac{|2,0\rangle - |0,2\rangle}{\sqrt{2}} \equiv |+45°,-45°\rangle, \qquad (3)$$

$$\Psi_0 = |1,1\rangle \equiv |x,y\rangle,$$

one obtains three states that can be transformed into one another by means of only retardation plates. Indeed, all three vectors of the new basis have $P=0$. They all correspond to pairs of correlated photons with orthogonal polarizations: right and left circular, linear at $\pm 45°$ to x, and along x and y. In this work, we experimentally realize transformations between these states.

The states Ψ_+, Ψ_-, Ψ_0 have much in common with quantum ternary logic states (''trits'') suggested in [10]. Indeed, the Bell states $1/\sqrt{2}(|H\rangle|H\rangle \pm |V\rangle|V\rangle)$ of [10] correspond to $\Psi_\pm$; the Bell state $1/\sqrt{2}(|H\rangle|V\rangle + |V\rangle|H\rangle)$ corresponds to Ψ_0, and the Bell state $1/\sqrt{2}(|H\rangle|V\rangle - |V\rangle|H\rangle)$ makes no sense in the case of indistinguishable photons. It is essential that, unlike the states introduced in [10], all states considered here relate to a single spatial mode. This has an important practical advantage, since it removes the requirement of equalizing optical paths, which was necessary in [10].

In the experiment (Fig. 1), we use a type-I lithium iodate

crystal pumped by a cw He-Cd laser with wavelength 325 nm and vertical polarization. The pump is split into two colinear beams, so that horizontally polarized SPDC with $\mathbf{e} = (1,0,0)$ is generated in two spatially separated domains. The pump radiation is suppressed by the cutoff filter F. After the crystal, the SPDC radiation from one of the domains is passed through a $\lambda/2$ plate oriented at 45° to the initial polarization. The corresponding biphoton polarization vector becomes $(0,1,0)$. Both SPDC beams are then joined together by means of a polarizing beam splitter PBS1. In fact, this part of the setup is a Mach-Zehnder interferometer, with the nonpolarizing beam splitter for the pump at the input and a polarizing beam splitter for biphoton radiation at the output. After the interferometer, the state is

$$\Psi = \frac{1}{\sqrt{2}}(|2,0\rangle + e^{i\phi}|0,2\rangle), \qquad (4)$$

where the phase ϕ is varied by means of a piezoelectric element (PE) shifting the mirror at the input of the beam splitter. Preparation of the biphoton state is accomplished by introducing a retardation plate RP (either a half-wave plate or a quarter-wave plate) after the beam splitter.

A half-wave plate with the optic axis oriented at an angle χ to the horizontal direction transforms [5] the state (4) into the state of the form (1) with

$$|c_1|^2 = |c_3|^2 = \frac{1 - \sin^2 4\chi \sin^2 \frac{\phi}{2}}{2}, \qquad |c_2|^2 = \sin^2 4\chi \sin^2 \frac{\phi}{2}. \qquad (5)$$

At $\phi = \pi$ and $\chi = \pi/8, 3\pi/8, \ldots$, $|c_1| = |c_3| = 0$, i.e., the state (4) is completely transformed into the state $|1,1\rangle$. In our notation, this is the transition $\Psi_- \rightarrow \Psi_0$. Note that, if $\phi=0$, the state (4) is invariant with regard to the action of a half-wave plate, $\Psi_+ \rightarrow \Psi_+$. Similarly, for a quarter-wave plate oriented at the angle χ,

$$|c_2|^2 = \sin^2 2\chi \left(\cos \frac{\phi}{2} + \cos 2\chi \sin \frac{\phi}{2} \right)^2, \qquad (6)$$

and the transformation from the state (4) to the state $|1,1\rangle$ is achieved at $\phi=0$, $\chi = \pi/4$. This describes the transition $\Psi_+ \rightarrow \Psi_0$. At the same time, a quarter-wave plate with $\chi = \pi/4$ leaves Ψ_- invariant.

Transitions from the states Ψ_- and Ψ_+ to the state Ψ_0 can be demonstrated experimentally by measuring the second-order correlation function of the final state,

$$G_{xy}^{(2)} \equiv \langle \Psi | E_x^{(-)} E_y^{(-)} E_x^{(+)} E_y^{(+)} | \Psi \rangle, \qquad (7)$$

where $E_{x,y}^{(\pm)}$ are field operators for the modes x and y. Indeed, for a state of the form (1), we have $G_{xy} \sim |c_2|^2$. The correlation function G_{xy} is measured by means of a polarizing beam splitter PBS2, two photodetectors, D1 and D2, and a coincidence circuit CC (Fig. 1). The pinhole PH with diameter 1 mm and the interference filter IF with full width at half maximum $\Delta\lambda = 10$ nm and central wavelength $\lambda = 650$ nm are
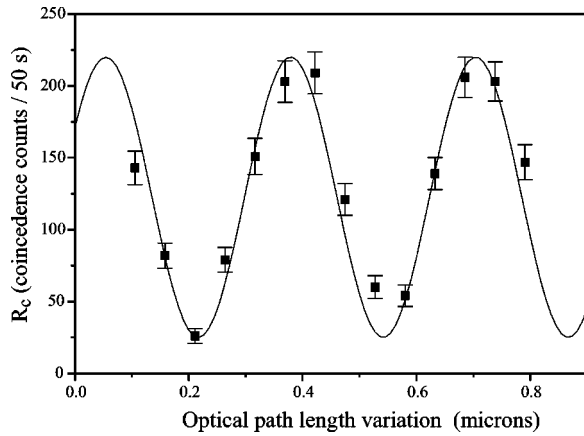
FIG. 2. Coincidence counting rate $R_c \sim G_{xy} \sim |c_2|^2$ as a function of the optical path length variation (phase ϕ in Fig. 1). The $\lambda/2$ plate after the Mach-Zehnder interferometer is oriented at $\pi/8$. Maxima of the dependence correspond to the Ψ_- state formed at the output of the interferometer; the half-wave plate transforms it into $|1,1\rangle$. In the minima, the state at the output of the interferometer is Ψ_+, and it is invariant to the action of the half-wave plate.

used for the spatial and frequency selection of the SPDC colinear frequency-degenerate radiation. The coincidence counting rate R_c, which is proportional to G_{xy}, is measured either as a function of the optical path length variation (phase ϕ variation) introduced by the piezoelectric element or as a function of the retardation plate orientation (angle χ variation).

The experimental dependencies obtained with the half-wave plate are shown in Figs. 2 and 3. First, we fix the orientation of the plate $\chi = \pi/8$ and measure R_c as a function of ϕ, which is determined by the voltage applied to the piezoelectric element (Fig. 2). In the minima, the state at the output of the interferometer is Ψ_+, which stays the same after the half-wave plate. At the maxima, the interferometer creates the state Ψ_-, which is then transformed into Ψ_0 by the half-wave plate. Fixing the phase ϕ at a maximum ($\phi = \pi$), we measure the dependence of R_c on the half-wave plate angle χ [Fig. 3(a)]. The high coincidence counting rate at the maxima of this dependence (in comparison with the accidental coincidence counting rate, which is less than $0.1 \ \mathrm{s}^{-1}$) indicates that the state $|1,1\rangle$ is formed. However, to check that Ψ_- is fully transformed into Ψ_0, we need to measure the correlation functions G_{xx} and G_{yy}, which are proportional to $|c_1|^2$ and $|c_3|^2$, respectively. Such measurements are performed by introducing an additional block before the polarizing beam splitter PBS2. This block (framed by a dashed line in Fig. 1) includes a polarizer selecting x or y polarization and a half-wave plate rotating the polarization by $\pi/4$. With this block introduced into the setup, R_c is proportional to $G_{xx} \sim |c_1|^2$ or $G_{yy} \sim |c_3|^2$, depending on the polarizer orientation. For instance, Fig. 3(b) shows the dependence of $|c_1|^2$ on χ for the phase ϕ being the same as for Fig. 3(a). One can see that at the angles χ where maxima of $|c_2|^2$ are observed [Fig. 3(a)], the amplitude $|c_1|$ (and, similarly, $|c_3|$) is almost completely suppressed [Fig. 3(b)]. The background coincidence counting rate in Fig. 3(b) (the visibility of the interference pattern is 90%) can be explained by nonequal losses for the states $|2,0\rangle$ and $|0,2\rangle$.

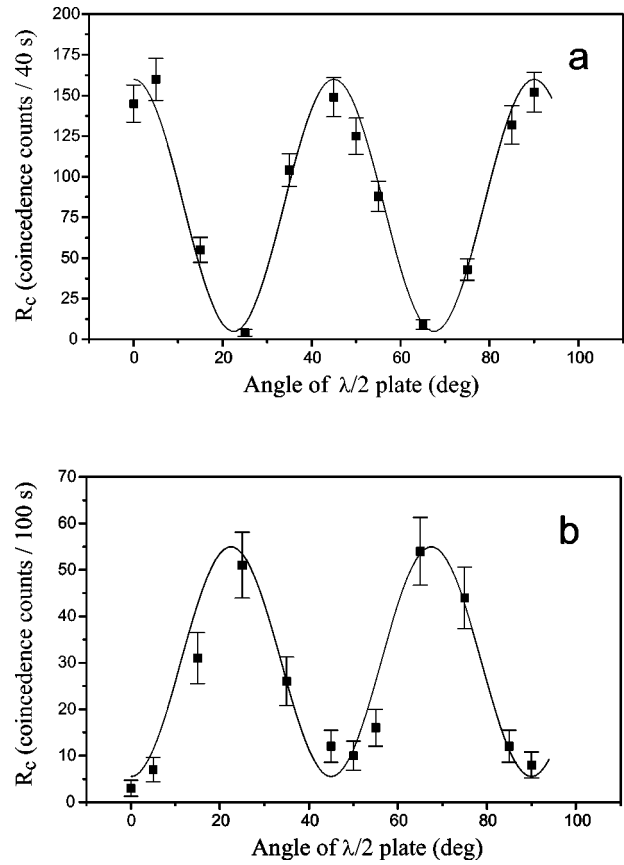Similarly, to perform the transformation $\Psi_+ \rightarrow \Psi_0$, one

FIG. 3. Coincidence counting rate corresponding to (a) G_{xy} and to (b) G_{xx} as a function of the angle χ of the $\lambda/2$ plate. In the lower case, the framed block in Fig. 1 is inserted. For both dependencies, the phase ϕ introduced by the piezoelectric element is π, i.e., the state at the output of the interferometer is Ψ_-.

should use a quarter-wave plate as the retardation plate in Fig. 1. The phase ϕ in this case should be equal to 0. In Fig. 4, the dependence of $G_{xy} \sim |c_2|^2$ on χ at $\phi = 0$ is shown. In accordance with Eq. (6), the period of this dependence is twice larger than in the case of the half-wave plate.

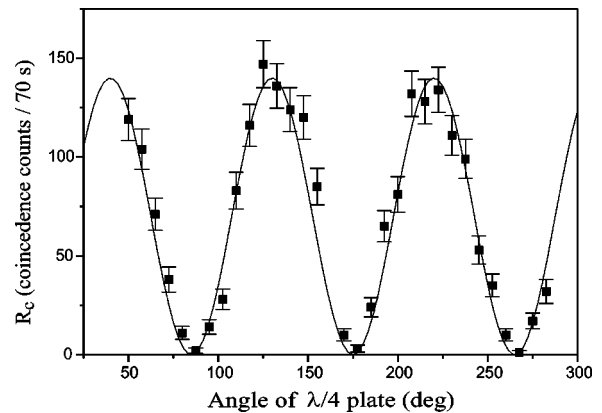All dependencies shown in Figs. 2–4 demonstrate nonclassical interference with high visibility. If both biphoton

FIG. 4. Coincidence counting rate corresponding to G_{xy} as a function of the angle χ of the $\lambda/4$ plate. The phase ϕ introduced by the piezoelectric element is 0, i.e., the state at the output of the interferometer is Ψ_+. In the maxima, the plate transforms it into the state Ψ_0; in the minima, it leaves it invariant.

states $|2,0\rangle$ and $|0,2\rangle$ generated in separate spatial domains are projected onto a single polarization direction [11], one can observe interference in coincidences, regardless of the delay introduced between the SPDC beams. In principle, the crystal inside the interferometer can be replaced by two separate crystals, placed at different distances from the beam splitter [12]. The only condition for the interference is that the arms of the interferometer should not differ by more than the pump coherence length. This property is due to the colinear degenerate phase matching used in our experiment. In a similar interference experiment with noncolinear SPDC performed previously [13], equality of the optical path lengths for two crystals was required [14].

Another paradoxical feature of this experiment should be pointed out. The state $|1,1\rangle$, which is what one calls ''a type-II biphoton,'' is produced by two independent ''type-I biphoton states'' $|2,0\rangle$ and $|0,2\rangle$. At the same time, the biphoton flux is so low (about hundreds of s^{-1}) that biphotons, if considered as ''wavepackets'' with coherence length $l_{coh} = \lambda^2/\Delta\lambda \sim 40\mu$, almost never overlap. This shows that, un-like single photons, biphotons should not be viewed as independent wavepackets [5,15].

Thus, we have demonstrated switching among the three states Ψ_-, Ψ_+, and Ψ_0: the transitions $\Psi_- \to \Psi_0, \Psi_+ \to \Psi_0$ are performed by half-wave and quarter-wave retardation plates, respectively. Note that the transition $\Psi_- \to \Psi_+$ can be performed by introducing a π phase shift between x- and y-polarized light, i.e., by inserting a half-wave plate with the axes parallel to the x,y directions. It is worth noting that all these transformations are reversible.

A remarkable property of retardation plates is that they leave invariant the number of biphotons, i.e., do not split photon pairs. This could be used for developing ''biphoton'' communication systems where biphotons propagate along a single direction (for instance, in an optical fiber) and are transformed by retardation plates.

[1] I. L. Chuang and Y. Yamamoto, Phys. Rev. A **52**, 3489 (1995).

[2] B. Schumacher, Phys. Rev. A **51**, 2738 (1995).

[3] T. Sleator and H. Weinfurter, Phys. Rev. Lett. **74**, 4087 (1995).

[4] N. J. Cerf, C. Adami, and P. G. Kwiat, Phys. Rev. A **57**, R1477 (1998).

[5] A. V. Burlakov and D. N. Klyshko, Pis'ma Zh. Éksp. Teor. Fiz. **69**, 839 (1999) [JETP Lett. **69**, 796 (1999)].

[6] To be exact, the state of light generated via SPDC is given by a superposition of vacuum and two-photon states. However, in many experiments, including the one discussed here, the vacuum state makes no contribution.

[7] A. Sudbery, *Quantum Mechanics and the Particles of Nature* (Cambridge University Press, Cambridge, UK, 1986).

[8] D. N. Klyshko, Zh. Éksp. Teor. Fiz. **84**, 1955 (1997) [JETP **84**, 1065 (1997)].

[9] The explicit form of G in this case is given in [5].

[10] K. Mattle *et al.*, Phys. Rev. Lett. **76**, 4656 (1996).

[11] A. V. Burlakov *et al.*, Pis'ma Zh. Éksp. Teor. Fiz. **69**, 788 (1999) [JETP Lett. **69**, 831 (1999)].

[12] A related method using separated crystals was demonstrated recently by P. G. Kwiat *et al.*, Phys. Rev. A **60**, R773 (1999), who studied polarization correlations between type-I biphotons in the case of noncolinear phase matching.

[13] Z. Y. Ou *et al.*, Phys. Rev. A **41**, 566 (1990).

[14] Another way to avoid equalizing optical path lengths is to join radiation from different crystals in the far-field zone, instead of using a beam splitter; see A. V. Burlakov *et al.*, Phys. Rev. A **56**, 3214 (1997).

[15] D. V. Strekalov, T. B. Pittman, and Y. H. Shih, Phys. Rev. A **57**, 567 (1998).