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Two-Photon Spectron

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The propagation of a two-photon light in a transparent medium with group velocity dispersion is considered. It is shown that, even in the stationary case of two-photon light generation by cw pumping, the second-order light correlation function behaves like a short pulse: when propagating in a medium, this function smears and at large distances acquires the spectral shape of two-photon radiation. *© 2002 MAIK "Nauka/Interperiodica".*

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Two-photon light is ordinarily obtained in experiments with spontaneous parametric scattering (SPS) [1]; it is of considerable interest in the context of generating so-called entangled states in optics. At present, the use of two-photon light for quantum information transmission is the subject of animated discussion [2].

In the simplest form, the state vector of radiation generated in SPS can be written, with allowance for the polarization, as $|\Psi_{\rm I}\rangle = |{\rm vac}\rangle + c|2, 0\rangle$ for the type-I matching and $|\Psi_{\rm II}\rangle = |{\rm vac}\rangle + c|1, 1\rangle$ for the type-II matching. In these expressions, $|n, m\rangle$ denotes the state with n photons in the polarization mode x and m photons in the polarization mode y; the parameter $c \ll 1$ specifies the amplitude of a two-photon state, and $|{\rm vac}\rangle$ stands for the vacuum state. However, this expression is rather idealistic: in reality, the spectrum, both frequency and angular, of a two-photon light is always of a finite length. For example, in the case of frequency-degenerate matching, the state generated in the SPS from cw pumping has the form of spectral decomposition [3]:

$$
\begin{aligned}
|\Psi\rangle &= |{\rm vac}\rangle + c\int d\Omega F(\Omega) a_s^\dagger\left(\frac{\omega_p}{2} - \Omega\right) a_i^\dagger\left(\frac{\omega_p}{2} + \Omega\right)|{\rm vac}\rangle \\
&\equiv |{\rm vac}\rangle + c\int d\Omega F(\Omega)\left|\frac{\omega_p}{2} - \Omega\right\rangle_s \left|\frac{\omega_p}{2} + \Omega\right\rangle_i,
\end{aligned}
\tag{1}
$$

where ω_p is the pump frequency, and the indices i and s correspond to the idler and signal modes, respectively. These may be the polarization (for the type-II matching) or spatial modes. The amplitude $F(\Omega)$, usually called the biphoton amplitude,[1] determines the spectral properties of a two-photon light. It has different forms for the type-II and type-I matchings:

$$
\begin{aligned}
F_{\rm II}(\Omega) &= \frac{\sin(DL\Omega/2)}{DL\Omega/2}, \\
F_{\rm I}(\Omega) &= \frac{\sin(D''L\Omega^2/2)}{D''L\Omega^2/2},
\end{aligned}
\tag{2}
$$

where L is the length of nonlinear crystal, D is the difference in the reciprocal group velocities of the signal and idler photons in the nonlinear crystal, and D'' is the second derivative of the dispersion relation $k(\omega)$ in the nonlinear crystal. One can see from Eq. (1) that, in the presence of spectral distribution, the light emitted in SPS always occurs in the entangled state.

The spectrum of a two-photon light in the vicinity of the degenerate phase-matching frequency $\omega_p/2$ is determined by the square of the modulus of the spectral amplitude $F(\Omega)$. Accordingly, the first-order correlation function has the form

$$
G^{(1)}(\tau) = 4|c|^2 \exp\left\{-i\frac{\omega_p}{2}\tau\right\}\int d\Omega |F(\Omega)|^2 \cos(\Omega\tau). \tag{3}
$$

For the second-order correlation function, calculations give the following expression:

$$
G^{(2)}(\tau) = 4|c|^2\left|\int d\Omega F(\Omega)\cos(\Omega\tau)\right|^2. \tag{4}
$$

For SPS in crystals with a length on the order of 1 cm, the typical width of the second-order correlation function is equal to several tens or hundreds of femtoseconds.

Let us now consider the propagation of two-photon light in a transparent dispersion medium. In the vicinity of degenerate matching, the dispersion relation in this medium can be written as $k(\omega) = k(\omega_p/2) + k'(\omega_p/2)(\omega - \omega_p/2) + k''(\omega_p/2)(\omega - \omega_p/2)^2/2$. It is well known that the

[1] A biphoton is referred to as a pair of photons with correlated moments of creation, frequencies, wave vectors, and polarizations.

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third term in this expansion is responsible for the smearing of short pulses in the medium. For the extended dispersion medium, $z \gg l_d$, where the dispersion length can be defined as $l_d = \tau_0^2/2pk''$ and τ_0 is the initial pulse duration, the pulse acquires a shape coinciding with its spectrum. Such a pulse has come to be known in the literature as a "spectron" [4].

Quite the same effect arises for two-photon light propagating in a dispersion medium. In this case, the creation operators $a^+_{s,i}(\omega_p/2 \pm \Omega)$ in Eq. (1) assume frequency-dependent phase factors, which can be interpreted as the appearance of a factor $\exp\{i(k_i'' + k_s'')\Omega^2 z/2\}$ for the spectral amplitude $F(\Omega)$. As a result, the first-order correlation function, as well as the spectrum, does not change. However, the second-order correlation function (4), which contains $F(\Omega)$ instead of $|F(\Omega)|^2$ under the integral sign, changes. Since the relation between $F(\Omega)$ and $G^{(2)}(\tau)$ is analogous to the relation between the pulse spectral amplitude and the square of the pulse envelope, the second-order correlation function behaves like a short pulse propagating in a dispersion medium. For $z \gg l_d$ (as in the femtosecond pulse optics, this condition may be called "far zone" condition), the correlation function has the form

$$G^{(2)}(\tau) \sim |F(\Omega)|^2\big|_{\Omega = \tau/k''z},$$

where $k'' = k_s'' + k_i''$. As in the case of a short pulse, the width of the correlation function after passing through the dispersion medium of length z becomes $\tau = 2\pi z k''/\tau_0$, where τ_0 is its initial width. Therefore, if the initial width of the second-order correlation function is 50 fs, its width becomes equal to 6 ns after passing two-photon light through an optical fiber 1 km in length (it is assumed that k'' for the fiber equals 3×10^{-28} s^2/cm [4]). As for the shape of the correlation function, it coincides with the spectrum given by Eq. (2). Such a two-photon wave packet in the far zone can be called *a two-photon spectron.*

The smearing of the biphoton correlation function in a dispersion medium should necessarily be taken into account when designing the schemes of quantum information transmission by two-photon light. It should be noted that this smearing in optical fibers can be compensated using the known linear methods of pulse compression [5] (the nonlinear methods are unsuitable because of the low intensity of biphoton fields). One sometimes assumes erroneously that the shape of the second-order correlation function for biphoton light manifests itself in the so-called anticorrelation effect [7], which consists of a sharp decrease (practically to zero) in the number of coinciding photocounts of two detectors, which detect both signal and idler beams (before detection, the signal and idler beams impinge on a beam splitter, so that the effect can only be observed if the optical paths of the signal and idler photons are balanced before beam splitting). However, it is known that the presence of a dispersion medium, through which the signal and idler beams propagate before beam splitting, has no effect on the shape of the anticorrelation "dip" [7]. This effect can easily be explained if one considers that the dip shape is associated not with the second-order but with the first-order correlation function [8]. However, according to Eq. (3), the propagation of two-photon light (as well as any other radiation) in a transparent medium with group velocity dispersion does not affect the shape of the first-order correlation function.

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