Superbunched light in a feedback loop with random properties

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Abstract

The statistical properties of light in a novel type of electrooptic feedback loop are studied both theoretically and experimentally. The feedback under consideration is closed via the photodetector shot-noise and has, therefore, a random character. The random fluctuations of the feedback factor lead to enormous fluctuations of light intensity, i.e., to the generation of superbunched light in the feedback loop. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

Feedback is an important part of every amplifier, stabiliser, or generator. That is why different radiotechnique devices with a feedback loop [1,2] as well as mathematical models of linear and nonlinear self-oscillation systems [3,4] are studied in detail. However, usual radiophysics theories can be insufficient for explaining the properties of light in certain electrooptic devices with a feedback loop, which have been studied during the last years [5–20].

The possibility of light transformation in a feedback loop is considered since 1986 when Yamomoto et al. [5] and Mashida and Yamomoto [6] obtained the reduction of photocurrent fluctuations below the shot-noise limit by means of an electrooptic negative feedback. This result was repeated by Fofanov [7–9] in 1988. The authors of Refs. [10,11] have shown that the Heisenberg uncertainty relation for light can be changed by the feedback loop. Masalov et al. confirmed this hypothesis experimentally in Ref. [12], where the photocurrent fluctuations were reduced below the standard quantum limit. In Ref. [11], the term ‘supersqueezed’ was proposed for such light.

Such a detailed theoretical analysis of the quantum properties of light is possible only for linear negative feedback, where all fluctuations are small and one can use the spectrum analysis. In more complicated nonlinear cases such as optical systems with multiplicative noise [17–19] or optical turbulence [20], only a classical analysis based on the mathematical theories of nonlinear self-oscillation [3,4] or the dynamic chaos [21] is possible.

In the present paper, we study both theoretically and experimentally the statistical properties of light in a feedback loop with unusual features: the feedback is closed via the photodetector shot-noise and, hence, has a random character. We shall show that in this feedback loop, light has superbunched properties with anomalously large intensity fluctuations.

This paper is organized as follows. In Section 2, we consider the main features of our feedback and show that the feedback factor is a random variable. Section 3 is devoted to the linear analysis of light properties in the feedback loop. Such analysis leads to a contradiction, which can be solved only by taking into account the feedback nonlinearity. The Fokker–Planck equation for the light intensity distribution function in the nonlinear case is obtained and solved in Section 4. The experimental results are considered in Section 5. Finally, we discuss the main properties of light and electric signal in the system under consideration.

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2. Main features of the feedback loop with random character

Consider the scheme in Fig. 1. The laser beam is diffracted by an ultrasonic wave in the acousto-optical modulator (AOM). The diffracted beam with the intensity \( I(t) \) is fed to a photomultiplier tube (PMT). The photocurrent \( i(t) \) is amplified by a resonance amplifier (RA) with the central frequency \( \Omega_0 \) and the bandwidth \( \Delta \). The electric signal \( w(t) = w_0(t)e^{\Omega_0 t} \) excites the ultrasound in the AOM. This feedback differs from usual systems \(^{11,12}\) by the absence of a reference beam on the PMT.

The properties of the amplitude of the diffracted light beam \( E(t) \) are defined by the properties of the electric signal \( w(t) \): \( E(t) = eE_p w_0(t) \), where \( E_p \) is the amplitude of the laser radiation and \( e \) is defined by the diffraction efficiency; i.e., the optical frequency \( \omega \) of the diffracted light shifted by \( \Omega_0 \), and the amplitude of light depends on the amplitude of the electric signal \( w_0(t) \). The PMT photocurrent is proportional to the diffracted light intensity \( i(t) = \beta I(t) = \beta e^2 I_p |w(t)|^2 \). Therefore, the fluctuation spectrum of the photocurrent depends on the intensity fluctuation spectrum of the electric signal \( |w(t)|^2 \). However, the width of this spectrum is defined by the bandwidth of RA \( \Delta \ll \Omega_0 \). Thus, we can see that the photocurrent fluctuations have frequencies much less than the central frequency \( \Omega_0 \) and cannot be amplified by the RA. On the other hand, in the experiment the feedback is closed. How can it be?

To eliminate this contradiction we must find a source of photocurrent fluctuations at frequencies near \( \Omega_0 \). This source is the PMT shot-noise, i.e., the discrete pulse structure of the photocurrent. The shot-noise is a random character with an exponential distribution function, and the variance of the feedback factor is equal to its mean value. Fluctuations of this feedback factor cause enormous fluctuations of the light intensity in the feedback loop, i.e., lead to the generation of superbunched light. Now we formulate the mathematical model of our feedback loop in order to express the mean value of the feedback factor via the parameters of the AOM, PMT and RA.

3. Theoretical analysis of the feedback loop in the linear approximation

3.1. Mathematical model: mean value of the feedback factor

One can describe all processes in the feedback loop in terms of the three functions \( I(t) \), \( i(t) \), \( w(t) \). It is simple to write connections between them using the classical statistical optics as listed below.

(1) The diffracted electric field \( E(t) = eE_p w_0(t)e^{\Omega_0 t} \). Then

\[
I(t) = e^2 I_p |w_0(t)|^2 ,
\]  
where \( I_p \) is the intensity of the laser beam.

(2) The Fourier transformation of the electric signal

\[
w(\Omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} w(t)e^{i\Omega t} dt = K(\Omega)e^{i\Omega t} ,
\]  
where

\[
g_2 = \frac{\langle I^2 \rangle}{\langle I \rangle^2} ,
\]  
which is equal to unity for coherent light and to two for thermal light \(^{22}\). Light with \( g_2 < 1 \) (\( > 2 \)) is called antibunched (superbunched) light. We shall show that light in the considered feedback loop has superbunched character with \( g_2 \gg 2 \).

First let us discuss the properties of the diffracted light if the PMT is illuminated by coherent light. In this case, the electric amplitude \( w(t) \) has Gaussian character because the RA amplifies the stationary white shot-noise \(^{22}\). Therefore, the diffracted light has thermal statistics \(^{23-25}\). Thus, our electrooptic loop transforms light with nonfluctuating intensity into thermal light with \( g_2 = 2 \). Then we can say that the feedback factor has a random character with an exponential distribution function, and the variance of the feedback factor is equal to its mean value. Fluctuations of this feedback factor cause enormous fluctuations of the light intensity in the feedback loop, i.e., lead to the generation of superbunched light.

Fig. 1. The scheme of the feedback loop under consideration: AOM – acousto-optical modulator; PMT – photomultiplier tube; RA – resonance amplifier.
where \( \tau_k \) denotes the delay time in the electrooptic loop (it is convenient to take into account the full delay time in this equation). Let the transmission function of the RA, \( K(\Omega) \), be Lorentzian:

\[
K(\Omega) = K_0 \frac{\Delta^2}{4(\Omega - \Omega_0)^2 + \Delta^2}.
\]  

(4)

(3) At last, the photocurrent consists of photocurrent pulses:

\[
i(t) = \sum_{j=1}^{n} H(t - t_j), \quad 0 \leq t_j \leq T
\]  

(5)

where \( H(t) = A \exp(-t^2/2\tau_0^2) \) is the form of a single pulse and \( t_j \) is a random moment of the pulse appearance. Suppose \( T \) is less than the typical time scale of intensity fluctuations \( \Delta^{-1} \), then we can use the Poissonian distribution function for a number of photocurrent pulses \( n \) appearing during the time interval \( T \):

\[
P_T(n) = e^{-\beta TI(0)} \left( \beta TI(t) \right)^n / n!.
\]  

(6)

By \( \beta \) we denote the efficiency of the PMT. One can express any moments of the photocurrent in terms of light intensity units. The stationary solution of Eq. (8) is

\[
I(t) = \Delta C \int_0^\infty \left( I(t - \tau_L - \tau) + I_0 \right) e^{-\Delta t} d\tau,
\]  

(8)

where

\[
\Delta C = \frac{\pi}{4} (eAK_0)(\beta I_0 \tau_0)(\tau_0 \Delta) e^{-\Delta^2 / 4\tau_0^2}
\]  

(9)

is the mean value of the feedback factor, and \( I_0 \) is the total value of the additive noise caused by all feedback components in light intensity units. The stationary solution of Eq. (8) is

\[
I = \frac{C}{1 - C I_0}, \quad C < 1,
\]  

(10)

i.e., the feedback amplifies the noise of the RA, the PMT dark noise and the light of external sources that are fed to the PMT. Now, in order to analyse the light intensity fluctuations in the feedback loop, we must take into account the randomness of the feedback factor.

3.2. Insufficiency of the linear approximation: infinite moments

In Eq. (8), the light intensity in the time interval from \( t - \tau_L - \Delta^{-1} \) to \( t - \tau_L \) has main influence on the light intensity at the moment \( t \). However light intensity cannot change noticeably during this time interval, because the width of the fluctuation spectrum for the light intensity is less than \( \Delta \). Therefore, we can rewrite Eq. (8) approximately as a recurrent relation,

\[
I(t) = \xi \left( I(t - \tau_L) + I_0 \right).
\]  

(11)

where \( \xi(t) \) is a random feedback factor with an exponential distribution function, defined by the mean value \( C \):

\[
P(\xi) = C^{-1} e^{-\xi/C}.
\]  

(12)

For such a relation, it is simple to obtain the equation for the light intensity distribution function \( P(I) \):

\[
P(I) = \int_0^\infty P(y) \exp \left( -\frac{I}{C(y + I_0)} \right) dy.
\]  

(13)

Solving this equation is complicated; however, Eq. (13) gives the possibility to determine the moments of the light intensity. Multiplying both sides of Eq. (13) by \( I^n \) and integrating with respect to \( I \) and \( y \), we get

\[
\langle I^n \rangle = n! C^n \left( I + I_0 \right)^n.
\]  

(14)

The expression (10) can be derived from Eq. (14) for \( n = 1 \), which has physical meaning (\( \langle I \rangle > 0 \)) for the mean value feedback factor \( 0 < C < 1 \). At \( C \geq 1 \), the feedback loop operates as a generator rather than an amplifier, and it is necessary to take into account the nonlinearity of the feedback loop. Similarly, for \( n = 2 \) one can obtain the second normalised moment of light intensity as a function of \( C \),

\[
g_2(C) = \frac{2(1 - C^2)}{1 - 2C^2}.
\]  

(15)

According to this relation, light in the feedback loop has thermal character for weak feedback \( C \ll 1 \) [23–25], but its intensity fluctuations increase with the feedback factor and become infinite at \( C = 1/\sqrt{2} = 0.71 \). From Eq. (14), one can see that there is a term \( (1 - n! C^n)^{-1} \) in the expression for the \( n \)-th moment of the light intensity, i.e., this moment becomes infinite at \( C = 1/\sqrt{n!} \), which tends to zero at large \( n \). Hence for any \( C > 0 \) there exists \( n \) such that \( \langle I^n \rangle \) is infinite. In such situation, we must insert the nonlinearity into the model of the feedback loop for any
C, which limits intensity fluctuations. Only a nonlinear model can describe our feedback loop correctly.

4. Nonlinear description of the feedback loop: the Fokker–Planck equation

Now we change the light intensity under the integral in Eq. (8) to the nonlinear function $F(I)$, which cannot be greater than a certain value. This function describes the feedback nonlinearity,

$$ I(t) = \Delta \xi(t) \int_0^\infty \left\{ F(I + I_0) \right\} e^{-\tau t} \, d\tau. \quad (16) $$

Here $\xi(t)$ is a random feedback factor (Eq. (12)), which is delta-correlated because the shot-noise correlation time $\tau_n$ is shorter than other time scales of the feedback loop $\tau_L$ and $\Delta^{-1}$. Solving the nonlinear Eq. (16) is very complicated and cannot be obtained by direct methods.

Consider, therefore, the equation for the electric current $x(t)$ in the RA oscillatory circuit which is influenced by the random shot-noise:

$$ \ddot{x} + \Delta \dot{x} + \Omega_0^2 x = \eta(t). \quad (17) $$

Here, $\eta(t)$ is a random Gaussian delta-correlated function with variance $\langle \eta(t) \eta(t') \rangle = \Omega_0^2 \Delta CF (|x(t - \tau_L)|^2 + I_0)$. Using Eq. (17), one can obtain an integral equation for the time evolution of the squared current amplitude $|x(t)|^2$, which is equivalent to Eq. (16). On the other hand, properties of the random process $x(t)$ can be analysed by means of the Stratonovich method [3,4] for the investigation of the fluctuations in nonlinear self-oscillation systems. Rewrite Eq. (17) for the quadrature components $x_1(t)$ and $x_2(t)$, such that $x(t) = x_1(t) \cos(\Omega_0 t) + x_2(t) \sin(\Omega_0 t)$:

$$ \ddot{x}_1 + \frac{\Delta}{2} \dot{x}_1 = \eta_1(t), \quad \ddot{x}_2 + \frac{\Delta}{2} \dot{x}_2 = \eta_2(t). \quad (18) $$

where the random Gaussian noise sources $\eta_1(t)$ and $\eta_2(t)$ with zero mean values have the following correlation functions:

$$ \langle \eta_1(t) \eta_1(t') \rangle = \langle \eta_2(t) \eta_2(t') \rangle = \frac{\Delta}{2} \Delta CF \left( |x_1^2 + x_2^2| \right) e^{-\tau_n} \delta(t - t') $$

$$ \langle \eta_1(t) \eta_2(t') \rangle = 0. \quad (19) $$

Suppose the delay time $\tau_L$ is negligible; then we can obtain the Fokker–Planck equation for the distribution function of quadrature components [3,4],

$$ \frac{\partial P(x_1, x_2)}{\partial t} = - \frac{2}{\Delta} \sum_{i,j=1}^2 \frac{\partial}{\partial x_i} \left\{ K_{i,j}^{(1)} P \right\} + \frac{1}{2} \sum_{i,j=1}^2 \frac{\partial^2}{\partial x_i \partial x_j} \left\{ K_{i,j}^{(2)} P \right\}. \quad (20) $$

\[ K_{i,j}^{(1)}(x_1, x_2) = - \frac{\Delta}{2} x_i, \]

$$ + \frac{1}{2} \sum_{j=1}^2 \int_{-\infty}^{\infty} \frac{\partial}{\partial x_j} \langle \eta(t) \eta(t + \tau) \rangle d\tau $$

$$ = \Delta \left\{ \frac{\Delta}{2} \sum_{j=1}^2 \left( \frac{\partial}{\partial x_j} \right)^2 \right\} \left\{ F(I + I_0) - 1 \right\}, $$

$$ K_{i,j}^{(2)}(x_1, x_2) = \int_{-\infty}^{\infty} \langle \eta(t) \eta(t + \tau) \rangle d\tau $$

$$ = \Delta \left\{ \frac{\Delta}{2} \frac{\partial}{\partial I} \right\} F(I + I_0) \delta_{ij}, $$

where $I = |x|^2 = x_1^2 + x_2^2$.

From the stationary solution of Eq. (20) we get the distribution function of light intensity

$$ P(I) = Q \exp \left[ -\frac{\Delta}{4} \frac{\partial}{\partial I} \int F(I + I_0) \right], \quad (21) $$

where $Q$ is a normalisation constant. As we shall see in the next section, the function $F(I)$ must be taken in the form $F(I) = I/(1 + I/I_o)$, which is bounded above by the constant intensity $I_s$. Using this function, we obtain

$$ P(I) = Q \left[ \frac{\exp(-I/I_o)}{I_0} \right]^{2/C}. \quad (22) $$

Thus we obtain the distribution function for the light intensity in the feedback loop. Now we can evaluate any moment of light intensity for any given feedback factor $C$.

![Fig. 2. Theoretical dependence of the second normalized intensity moment $g_2$ on the feedback factor C for different ratios $I_0/I_s$: (a) $10^{-3}$; (b) $3 \times 10^{-4}$; (c) $10^{-4}$](image)
and the ratio $I_d/I_s$. For example, the dependence of the second normalised moment on the feedback factor $C$ for different $I_d/I_s$ ratios is shown in Fig. 2. In the next section, we compare these theoretical results with the experimental data.

5. Experimental results: intensity correlation and photocounts statistics

5.1. Measurements of the mean value feedback factor $C$

The scheme of the experimental setup is shown in Fig. 3. The He–Ne laser beam is diffracted by the ultrasound in the AOM. The diffracted beam is fed to the PMT. The photocurrent is amplified by the broad-band amplifier (BA) and by the RA with the central frequency $\Omega_0 = 50 \text{ MHz}$ and the bandwidth $\Delta = 3 \text{ MHz}$. The amplified electrical signal excites the ultrasonic wave in the AOM. The feedback factor value is controlled by the system of crossed polarisers (CP).

The mean value of the feedback factor $C$ can be defined from the dependence of the diffracted light intensity $I_d$ on the light intensity $I_{in}$ at the PMT input in the opened feedback loop. The typical dependence $I_d(I_{in})$ is shown in Fig. 4. At low input intensity, $I_d$ is proportional to $I_{in}$. By definition, the feedback factor $C$ is equal to the proportionality coefficient in this region. Further, the feedback nonlinearity starts to limit the diffracted light intensity as input light intensity increases. This dependence is well described by the nonlinear function

$$I_d = CF(I_{in}) = C \frac{I_{in}}{1 + I_{in}/I_s},$$

(23)

which was used in the previous section. The slow decrease of $I_d$ at $I_{in} \gg I_s$ in Fig. 4(b) was not taken into account in the theoretical model.

5.2. Intensity correlation: the second moment $g_2$

Part of diffracted light in Fig. 3 was extracted from the feedback loop by means of a beam-splitter (BS). The properties of the intensity fluctuations of this light beam were studied by means of the Brown–Twiss intensity interferometer [26, 27] and by the method of photocounts statistics [28]. We used the Brown–Twiss interferometer (Fig. 3(II)), that consists of a beam-splitter, two PMTs and a coincidence scheme (with the characteristic time $T_C = 1 \text{ ns}$), for measuring the second normalised intensity moment

$$g_2 = \langle I^2 \rangle / \langle I \rangle^2 = \frac{N_3}{N_1 N_2 T_C},$$

(24)

where $N_1$, $N_2$ are the photocount rates at two PMTs and $N_3$ is the coincidence counting rate. The experimental dependence $g_2(C)$, shown in Fig. 5, corresponds well to the theoretical one, obtained from Eq. (22) for $I_{in}/I_s = 3.2 \times 10^{-4}$.

5.3. Photocounts statistics

In the previous sections, we saw that for the feedback factor $C \sim 1$, the second normalised moment for the light in the feedback loop $g_2 \gg 1$, i.e., the intensity variance is
much higher than its mean value. Then the intensity distribution function must have a very long ‘tail’. Such a tail can be observed by the method of the photocounts statistics [28]. To this aim, the output light beam was fed to an additional PMT (Fig. 3(III)). A fast gate (G) operated by CAMAC allows to measure the number of photocounts appearing during a short time interval \( \tau = 0.5 \mu s \sim \Delta^{-1} \).

The experimental distribution function of the photocount number \( P_s(n) \) is shown in Fig. 6 (solid dots). For comparison in the same graph we show the Poissonian distribution function corresponding to the coherent light (crosses), with the same mean value. One can see that the experimental function is constant approximately for \( 2 \leq n \leq 7 \) whereas the Poissonian function decreases very rapidly.

The theoretical photocount distribution function can be obtained from Eq. (22) using the Mandel formula [22]:

\[
P_s(n) = \int_0^\infty P(I)e^{-\mu t}(\sqrt{\mu t})^n \frac{e^{-\mu t}}{n!} dt.
\]

However, we have to take into account the dead time of the PMT, which is only six times less than the gate time \( \tau \). To obtain the effective distribution function \( P_{ef}(n) \), we must multiply the real distribution function by the dead-time matrix [28]:

\[
P_{ef}(n) = \sum_{k=n}^{\infty} p^n_k P_s(k),
\]

where the matrix \( p^n_k \) has the following properties:

\[
p^n_k = h^n_k = h^n_0 = 0 \quad \text{at} \quad n > k,
\]

\[h^n_1 = 1; \quad h^n_k = n h^n_{k-1} + h^n_{k-1},
\]

where \( \gamma = \tau_0/\tau = 0.16 \). The theoretical points (open circles) in Fig. 6 were obtained from Eqs. (22), (25)–(27), where \( C = 1, \; l_d/l_s = 10^{-4} \) and the PMT efficiency \( \beta \) was chosen such that the mean photocount number

\[
\pi = \sum_{j=0}^{\infty} j P_{ef}(j)
\]

was equal to the experimental one. From this graph one can see that the theoretical distribution function (22) describes the experimental long tail of the photocount distribution very well. This tail demonstrates the superbunched properties of light in the feedback loop: there is a considerable possibility to find any number of photocounts \( n > 7 \) for very small light intensity \( \pi = 0.01 \), i.e., the light intensity fluctuations are much higher than the intensity mean value. Observation of the large photocounts number \( n > 7 \) is impossible due to the dead-time effect.

6. Conclusion

In this paper we have studied the statistical properties of light in an electrooptical feedback loop, which is closed via the PMT shot-noise and has, therefore, a random character. It was found that this light has superbunched properties, i.e., the light intensity variance is much higher than the intensity mean value. The degree of superbunching can be controlled by the value of the feedback factor \( C \) (Eq. (9)), which can be easily changed experimentally using the system of polarizers (Fig. 3).

In the theoretical description, we have used the Stratonovich method [3,4], which can be applied for our feedback loop under the assumption of negligibly small delay time \( \tau_L \). This assumption is not valid since \( \tau_L \gg \Delta^{-1} \); nevertheless, the stationary solution of Eq. (20) for the light intensity distribution function describes very well the experimental results obtained by means of the Brown–Twiss interferometer and the photocount statistic method (Figs. 5 and 6). However, for studying the light intensity autocorrelation function one has to use an equation, that takes into account the delay time of the feedback loop.

Finally, note that the generation of superbunched light in the feedback loop can be explained by the classical statistical optics, i.e., superflashing is not a quantum
effect in contrast to squeezing [10,11] and antibunching [22].

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